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Building students' conceptual understanding of operations on fractions using manipulatives: A junior high school perspective

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ABSTRACT

Mathematics plays a key role because it is one of the important subjects within the foundation that constitute the core curriculum for basic and secondary education. Mathematics must therefore be taught in a way to engage learners to construct their knowledge, which helps them build conceptual understanding using modern teaching strategies. Fractions are the building blocks for a solid conceptual understanding of algebra and other concepts in mathematics, especially at the basic school. Therefore, teachers must use strategies that make lessons more realistic and practical, such as manipulatives. The purpose of the study was to use manipulatives to build students' conceptual understanding of the operations of fractions. The study design was quasi-experimental, with a pre-/post-test method used for data collection to assess the impact of the intervention design. A sample of 50 junior high school students was selected purposively for the study. The data was analyzed using SPSS v.26. The researchers concluded that using manipulatives in the teaching of fractions improved the students' performance and helped build their conceptual understanding of the operations of fractions.

Keywords: mathematics, fractions, manipulatives, conceptual understanding

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INTRODUCTION

In the majority of nations around the world, mathematics is one of the crucial foundational disciplines that make up the core curriculum for basic and secondary education (OECD, 2017). This is due to the fact that mathematics has been identified as one of the topics that promotes the growth of a person's original, inventive, analytical, and problem-solving abilities (Ministry of Education, National Council for Curriculum and Assessment [MoE, NaCCA], 2019). Math is a "facilitating" subject, according to research. In other words, learning mathematics is a prerequisite for learning other topics and pursuing a wide range of occupations (Maldonado et al., 2018). We can appreciate the value of mathematics by looking at how it is used in technology and in our daily life. Mathematics is the only discipline that acts as a significant unifying force among the numerous branches of research, and without it, knowledge of science frequently stays rudimentary (Moyer, 2001).

According to Berger et al. (2020), mathematicians are interested in number, shape, and space. They also like to categorize things and prove that a given phenomenon must, can, or actually cannot occur. To ground mathematics teachings in reality, contemporary experts advise stressing context-based mathematics instruction (OECD, 2017). To help the students understand how essential mathematics is to their lives, real-world events are introduced into the mathematics classroom (Barnes, 2005). Many researchers both locally and internationally support this strategy since it is consistent with the problem-solving method to teaching mathematics (Bartolini & Martignone, 2020).

Most word problems are context-based, and they frequently involve students in debates, brainstorming, group projects, and other student-centered teaching and learning activities (Andamon & Tan, 2018). While some see math word problems as merely arithmetic exercises, other students find word problems to be the most difficult and intricate. For instance, Mereku (2004) found that Ghanaian students who participated in trends in mathematics and science studies (TIMSS) in 2003 did well on routine issues but badly on non-routine ones. They were able to solve questions that needed computations and basic knowledge recall. This suggests that teachers should make their lessons more practical by using activities and real-world examples to help students develop their conceptual understanding.

Policymakers in Ghana's education sector are still working to guarantee that the country's mathematics instruction complies with international norms. This is the result of the paradigm shift from the conventional method of teaching mathematics to contemporary educational techniques and abilities that strengthen conceptual understanding (Nabie et al., 2013). The government has started a number of curriculum reviews and modifications to make sure the nation's educational system stays up with the fast-moving international trends (Abudu & Mensah, 2016; Adu-Gyamfi et al., 2016; Mereku, 2010). Understanding and using mathematical languages, symbols, and notations is one of the main goals of the mathematics curriculum. In order to fulfill the purpose of mathematics education at the elementary level and beyond, it is expected that they will develop communication and collaboration skills (CC), which are key abilities (MoE, NaCCA, 2019). According to Baah-Duodu et al. (2020), mathematics should be taught utilizing engaging, hands-on, and mind-on methods that students will enjoy and take to heart as a way of life. If teachers were able to stick to this schedule, it would guarantee that the students could think, reason, and communicate numerically. As a result, math lessons must involve group projects and conversations in the classroom to get students thinking about and solving real-world problems (Adu et al., 2017).

One of the fundamental abilities that must be cultivated in the mathematics curriculum is the understanding of fractions (Bouck et al., 2017). This is due to the fact that it is crucial for comprehending algebra, geometry, and other mathematical concepts. Comprehension fractions requires a thorough understanding of all the concepts they can be used to represent (Cramer et al., 2010). Any learner must have a conceptual understanding of fractions in order to be able to handle more difficult subjects in the high school curriculum (Niemi, 1996). Learners will probably struggle with procedural competency in these areas if they have trouble comprehending the several meanings of fractions. National Council of Teachers of Mathematics (NCTM) curriculum and evaluation standards encourage using tangible materials and other representations to help kids build their grasp of the fraction idea, according to Bouck et al. (2017). There is sufficient evidence to demonstrate the significance of using images effectively in fraction problems (Cramer & Henry, 2002). Unfortunately, manipulatives are rarely used in textbooks, and when they are, they usually just serve as area models (Hodges et al., 2008). This indicates that pupils frequently lack the opportunity to investigate fractions using a variety of models and the time necessary to link the visuals to the pertinent ideas. The use of physical instruments seems to be crucial for learning because it creates mental models in students' heads that help them understand fractions (Cramer et al., 2010; Lee et al., 2021).

Items known as manipulatives can be touched, moved, rearranged, and in other ways handled by kids. They appeal to numerous senses (Johnson-Smith, 2022). Manipulatives can be physically manipulated as well as digitally, therefore they are not just tangible items that we can handle with our hands (Agyei et al., 2022). The use of manipulatives in classroom education, according to National Council of Supervisors of Mathematics (NCSM), raises student achievement (Berger et al., 2020). Regardless of the level, the declaration contends that educators must deliberately include the use of tangible and digital manipulatives into their lesson plans (Moore, 2014). This was done to make sure that every student could become proficient in mathematics (NCSM, 2013), as referenced in (Moore, 2014). With the help of manipulatives, instructors may provide students the chance to interact, practice, and manipulate objects and resources to solve challenges (Larbi & Mavis, 2016). Because students actively participate in the process of discovery during the learning process, manipulative is constructivist (Bosson-Amedenu, 2017; Larbi & Mavis, 2016). Numerous studies carried out locally support the use of manipulatives as one of the finest methods for teaching mathematics (Alolga & Essel, n. d.; Boakye, 2019). Researchers have found that students who use manipulatives in some mathematics areas, such as algebra and fractions, among others, are more likely to succeed than those who do not have access to manipulatives (Chappell & Strutchens, 2001; Sebesta & Martin, 2004). Therefore, the goal of this research is to develop, through the use of manipulatives, students' conceptual knowledge of operations on fractions.

Statement of the Problem

As was already said, fractions serve as the foundation for a strong mental knowledge of algebra and other mathematical ideas, particularly in the basic school. Unfortunately, most junior high school pupils do not have a strong interest in fractions because they believe that fractional notions are difficult and impossible. According to data collected throughout time, a number of junior high school students in their last year have been denied admission to senior high schools in the majority of districts of Ghana's Ashanti Region because of their poor math scores on basic education certificate exam (BECE). Students' performance on the mathematics portion of BECE was negatively impacted by their lack of knowledge of fractional concepts and operations, according to data collected from classroom activities, homework, and end-of-term exams.

This information supports the claim made by Bingham and Rodriguez (2019) that fractions are among the least understood mathematical topics taught in elementary schools. Therefore, the study aims to develop students' conceptual comprehension of fractional operations through the use of manipulatives.

Objectives of the Study

1. To determine if manipulatives could build students' conceptual understanding of operations on fractions.
2. To determine the extent to which manipulatives could improve students' performance in fractions.

Research Questions

1. What is the impact of manipulatives on students' conceptual understanding of operations on fractions?
2. To what extent does the use of manipulatives to teach the concept of fractions help improve students' performance?

LITERATURE REVIEW

History of Manipulatives

Manipulatives as a notion have a long history (Caglayan, 2019; Nikiforidou, 2019). Physical objects have been employed by people from numerous cultures around the world to assist them in resolving common math issues. Middle East and ancient Southwest Asia both developed counting boards. Counting boards were trays made of wood or clay that were dusted with fine sand. In order to take an inventory or total an account, for instance, the user would draw symbols in the sand. The earliest abacus was made by the ancient Romans by altering counting boards (Boakye, 2019). Chinese abacus, which was used centuries later and may have been a modification of Roman abacus, was another. In Americas, similar devices were created. The counting implements used by Mayans and Aztecs included wires or maize kernels strung on strings over a wooden frame. The knotted cords known as "quipu" were the Incas' special method of counting. The first real manipulatives were created in the latter half of the 1800s. These manipulable items were made with teaching mathematical principles in mind and appeal to all of the senses of the human body (Bartolini & Martignone, 2020). Friedrich Froebel, a German educator, established the first kindergarten program in history in 1837. He created many tools to aid his kindergarteners in identifying patterns and appreciating the geometric shapes present in nature.

Maria Montessori, an Italian-born educator, contributed to the development of the notion that manipulatives are crucial to education in the early 1900s. She brought out a variety of resources to aid young children in preschool and primary school in discovering and learning fundamental concepts in arithmetic and other topics. Manipulatives have become crucial in elementary school mathematics instruction since the early 1900s. In reality, NCTM has long advocated the use of manipulatives while instructing students in mathematics across all grade levels (Jimenez & Stanger, 2017; Sulistyaningsih et al., 2017).

The instructor has traditionally served as the classroom's leader. The teacher's responsibilities were working from the front of the room, imparting knowledge on the day's subject, and giving out homework. However, this function is altering in the dynamic field of mathematics. According to Cain-Caston (1996), the instructor is now more of an observer while the pupils are actively engaged in their learning (Bingham & Rodriguez, 2019). Math class has evolved from the tedious pencil and paper task it once was to a pleasant and exciting activity that many kids now look forward to in the majority of classrooms across the nation and in Ghana. Even though the same ideas are being taught, it has changed from being a chore to something enjoyable. What altered the situation? The use of manipulatives and a hands-on learning approach is the solution (Agyei et al., 2022).

Relationship Between Manipulative Use & Educational Theory

By enabling pupils to progress from the concrete stage to abstract reasoning, manipulatives can aid in learning (Heddens, 1986; Ross & Kurtz, 1993). Experts have shown that learning happens in three stages. These are the phases of cognition, association, and autonomy (Dowling et al., 2018). This is in line with Brunner's constructivist theory, which contends that when faced with new concepts, all learners–regardless of age–go through three stages of learning. Enactive (action-based), iconic (image-based), and symbolism are these levels (language-based) According to Clements et al. (2022), students' ability to think mathematically is improved by the usage of manipulatives. Stein and Bovalino (2001) claim that

> "manipulatives can be valuable instruments for encouraging pupils to think and reason in deeper ways. Such manipulatives as pattern blocks, tiles, and cubes can help kids acquire solid, integrated understandings of mathematical concepts by giving them real opportunities to compare and manipulate amounts."

Students must integrate and connect various ideas in a variety of ways in order to develop a deep comprehension of mathematical concepts. This kind of comprehension is referred to as "integrated-concrete" knowledge by Clements (1999), as mentioned in (Clements et al., 2022). Students can connect ideas and integrate their knowledge to develop a thorough comprehension of mathematical concepts by using manipulatives effectively. Teachers play a critical role in assisting students in using manipulatives effectively so that they can easily move through all three phases of learning and achieve a thorough comprehension of mathematical ideas (Larbi & Mavis, 2016; Sulistyaningsih et al., 2017).

Advantages of Using Manipulatives

The use of manipulatives in the classroom has a long history because they make arithmetic ideas understandable to a wide range of students. Low achievers and pupils with learning difficulties can benefit from it (Kontas, 2016; Marsh & Cooke, 1996; Moore, 2014; Ruzic & O'Connell, 2001). Researchers have found that when students use manipulatives and are subsequently given time to reflect on their experiences, their learning in mathematics is improved and their anxiety in math is greatly decreased. Exploring manipulatives, particularly self-directed exploration, creates a stimulating learning environment and encourages students to have a positive outlook on learning (Cain, 2021; Cain-Caston, 1996; Heuser, 2000; Moch, 2002). This means that manipulatives contribute to the enjoyment, realism, and application of mathematics. This is a result of the hands-on, experiential learning activities that students engage in when using physical learning instruments (Baah-Duodu et al., 2020).

Studies have indicated that children who have the opportunity to work with manipulatives in particular mathematics courses, such as algebra and fractions, among others, are more likely to succeed than students who do not (Chappell & Strutchens, 2001; Sebesta & Martin, 2004). Empirical research have demonstrated that consistent use of manipulatives, particularly in the teaching and learning of mathematics, results in the following benefits:

1. Communicating mathematical ideas and concepts, discussing mathematical ideas and concepts, and connecting mathematical symbolism to real-world problems.
2. Working cooperatively and using divergent thought to come up with a number of solutions to difficulties.
3. Making presentations, using a range of mathematical symbols to convey issues and solutions, and taking responsibility for their learning experiences.
4. Growing self-assurance in their ability to solve mathematical problems using their own approaches without following the teacher's instructions (source: info@hand2mind.com).

How Manipulative Support Fractions Education and Learning?

By portraying mathematical concepts in various ways, manipulatives aid students in developing a conceptual knowledge of mathematics, claims (Shaw, 2002), which leads to a number of advantages. Manipulatives can offer visual representations of ideas, just as a picture can be worth a thousand words, assisting children in recognizing and comprehending the notion of fractions.

The use of manipulatives improves students' reasoning and communication skills at all grade levels. Utilizing manipulatives helps students retain and use the material in fresh contexts for problem-solving, which increases their comprehension of fractions ideas (Shaw, 2002). Again, Shaw (2002) asserts that one strategy to resolve students' misconceptions about fraction concepts is to teach fractions using manipulatives. Additionally, students actively participate in developing conceptual understanding through the use of manipulatives.

Additionally, the promotion of manipulatives is based on the learning theories of Piaget, Bruner, and Montessori, which state that knowledge is developed and built as pupils' progress from concrete experiences to abstract thought (McNeil & Jarvin, 2007).

Bruner's hypothesis is supported by using manipulatives, which has pupils begin learning new information by executing activities on tangible things.

By enabling pupils to transition from tangible experiences to abstract reasoning, manipulatives aid in learning (Heddens, 1986; Reisman, 1982; Ross & Kurtz, 1993). According to educational specialists, this learning occurs in three stages.

METHODOLOGY

A quasi-experimental design was used for the study for practical and ethical reasons. According to Collom (2021) and Shadish et al. (2002), a quasi-experimental design involves a non-random assignment of participants to conditions in a study. The type of quasi-experimental design adopted by the researcher is- a group pretest-posttest design. An intervention design was rolled out after the pre-test.

Population and Sample

The study population was the Ampabame Junior High School students in Atwima Kwanwoma District in the Ashanti Region of Ghana. The school has a numerical strength of 135, comprising 42 forms one student, 50 forms two students, and 43 forms three students. The sample size was 50, comprising the form two school students. The students comprised 30 males and 20 females. This research used the purposive sampling method to get the sample size. Purposively, form two students were selected because they have had more extensive tuition in fractions than the form one students. The form three students could not be used since they were busy preparing for their BECE. According to Etikan and Bala (2017), purposive sampling is a form of non-random sample method whereby the researchers use their judgment in choosing participants for the study.

Study Tool

The primary tools for gathering data were pre- and post-test. The tools were created for convenient data capture, interpretation, analysis, and organizing. The purpose of the pre-test, which consisted of 10 questions, was to identify the relative strengths and weaknesses of the students' understanding of or assimilation of the idea of fractions, particularly the operations on fractions. The survey included questions on adding and subtracting fractions with both common and unusual denominators. Additionally, it included fraction division and multiplication exercises. The purpose of the post-test tool, which comprised 10 test items, was to gauge how much knowledge the students had gained regarding the idea of fractions, particularly the operations on fractions. Thus, the effectiveness of the rollout of the intervention was evaluated using the post-test method. Questions on fractional addition, subtraction, multiplication, and division as well as their use in word problems were included in the test items.

Fraction bars were employed as manipulatives in the teaching and learning of addition, subtraction, multiplication, and division of fractions during the implementation of the intervention. Each group of five pupils had a number of fraction bars, including wholes, halves, thirds, fourths, and so on. The pupils were initially led by the manipulatives to investigate how the component parts fit together to form a whole. For instance, two one-halves placed side by side have the same length when combined. The kids were able to recognize that each portion is equal to one when a whole is divided into two pieces. In addition, four quarters that are placed side by side have the same overall length. The pupils recognized that each part, when divided into four equal pieces, constitutes one-fourth of the whole. Additionally, the pupils were able to recognize that two one-fourths was equivalent to one half. The kids gained an understanding of fractions as pieces of a whole via these activities.

Students were assisted in exploring and appreciating equivalent fractions while utilizing the manipulatives. For instance, when two-thirds and four-sixths were instructed to be placed side by side, students noticed that the two different fractions took up the same amount of space. They were able to recognize and comprehend that the two fractions were equal in this way. Students were helped in investigating and comprehending which fractions are greater among various fractions while utilizing the manipulatives. Students were instructed, for instance, to align fraction bars representing the one-fourth and one-sixth fractions. They noticed that a quarter was larger than a sixth in terms of size. They were able to recognize that one-fourth is greater than one-sixth when comparing the two fractions.

Fractional Addition and Subtraction

The fraction bars served as a guidance for the students in each group as they added and subtracted fractions. They started by learning how to add and subtract fractions with the same denominator. Adding one-eighth and three-eighths is an illustration. The pupils had the option of choosing one eighth of a fraction bar and three additional eighths of a fraction bar. When they fitted everything together, they found that the fraction bars were all the same, resulting in four pieces of one-eighth. The pupils discovered that one eighth plus three eighths equals four eighths by using the fraction bars. They were also able to recognize that one-eighth minus three-eighths yields the result as minus two-eighths when the operation's sign was altered. The pupils were provided additional examples to practice on.

Students once more investigated utilizing the fraction bars to add fractions with unusual denominators. Using their prior understanding of equivalent fractions, they were led. For instance, multiplying by half and quarter. One-half and two-fourths are the same size when placed side by side, therefore students were prompted by the bars to recognize this relationship. The students understood that the result of one-half plus one-fourth is three-fourths. The students were also able to recognize that one-half less one-fourth equals one-fourth when the operation's sign was changed to minus. More examples were used as practice by the pupils.

Division of Fractions

The students were helped to realize that multiplication, taken literally, refers to how many times a quantity is multiplied. They were instructed to experiment with utilizing the fraction bars to multiply fractions. for instance, four times one-half. The pupils were instructed to choose four one-half fraction bar pieces to represent one-half times four. They were able to calculate that they had four halves in total, which meant that multiplying four by one-half would result in four halves. The same was true when students were asked to multiply one-fourth by five. They were able to choose five sections of the fraction bar, which gave them five-fourths when added together.

Fractional Division

The fraction bars were once more used to guide the pupils as they investigated how to split fractions. The pupils were given instructions on how to recognize that division in this sense denotes sharing. For instance, the number of twos in four can be calculated by dividing it by two. To help them understand the idea of fractional division, the students were instructed to use the fraction bars. For instance, divide one-half by four. They understood that the question was asking, "how many one-fourths are in a half?" The pupils' task was to choose a one-half fraction bar and calculate how many one-fourth fraction bars would fit in the same space when placed side by side. They saw that a one-half bar and two pieces of one-fourth bar had the same length. On this basis, they were taught to recognize that the result of half divided by a fourth is the entire number two. The similar exercise was used with the pupils to arrive to the conclusion that three-fourths divided by one-eighth equals a whole number, which is six. The pupils were encouraged to perform further examples like this.

Data Gathering

The pre-presentation stage was the initial stage. To help communicate the information by inspiring the students to learn and value mathematics in practical tasks, the researcher gathered enough tools, techniques, and tactics. In addition to several materials and resources created to improve the usage of the manipulatives, lesson notes were also provided. The presentation stage served as the second face and was where the actual lessons were taught. In order to develop their knowledge, students were free to explore, browse, and alter teaching tools and other resources. To demystify the notion of fractions and make it more alive, realistic, and interesting, this took a variety of forms, including self-learning, group discussions, role playing, contests, and speed tests.

The post-test was given in the third and final phase to gauge how well the intervention had worked. The test items were created by the researcher, hence content and construct validity were achieved. The test questions were all modified from former West Africa Examinations Council (WAEC) examinations as well as TIMSS and other standardized test questions. The post-exam had 10 theory test questions that were to be answered in 45 minutes. The students' grasp of adding fractions with the same denominator and fractions with unusual denominators was tested with four questions. They were put to the exam on their knowledge of fractional subtraction by way of two questions. Additionally, there were two questions on division of fractions and two more on multiplication of fractions among the test items. After getting the questions, measures were taken to have some other students attempt the items so they could rephrase or delete as necessary. In addition, the problems were reviewed by a senior mathematics teacher from the Ampabame Junior High School. The test's reliability was assessed using the test-retest reliability method. The test yielded a reliability coefficient of r=0.86. Therefore, it is possible to consider the study's test to be reliable.

Each student's performance on both tests was entered into the statistical package for the social sciences (SPSS) version 22.0 software's data view in order to analyze the data gathered. Charts were utilized to answer the study's research questions once the data entered into the software were transformed into frequency counts and percentages. Mann Whitney's independent samples. The U test was also utilized to look into gender differences in test results between the pre- and post-tests.

Table 1. Frequency distribution for pre- & post-test scores

Marks	Frequency	
	Pre-test scores	Post-test scores
11-20	4	0
21-30	9	3
31-40	10	6
41-50	15	7
51-60	12	11
61-70	0	10
71-80	0	10
81-90	0	3
Total	**50**	**50**

RESULTS

The analysis of the results has been presented in order of the research questions.

Research Question One: What is the Impact of Manipulatives on Students' Conceptual Understanding of Operations on Fractions?

Tests conducted were the main avenue to answer research question one. The scores of the tests were presented as shown in **Table 1**. **Table 1** presents the raw scores obtained from the pre- and post-test. A close observation reveals that the post-test performance is better than the pre-test results.

In order to give a visual impression of the impact of the use of manipulatives on the students' understanding and performance, a histogram was also used to compare the students' performance for both the control and experimental modes. It is practically clear from **Figure 1** that none of the students scored above 60% in the pre-test, but the post-test had more than half of the students scoring 60% or more.

Furthermore, out of the 50 students who took the pre-test, 13 scored 30% or below, indicating very abysmal performance. A whooping 25, representing half of the students scored between 31% and 50% inclusive, and only 12 of the students could score 50% and above but not more than 60%. As outlined, the students' performance has increased tremendously mainly because of the manipulatives in teaching the operations on fractions. It is worth noting that out of the 50 students, only three scored 30% or below. This is quite an enormous improvement from the pre-test. Also, only six students fall in 31% and 40%, and seven in 41% and 50%. This means that out of the 50 students, 16 of them scored 50% or below, and the rest scored a mark of more than 50%. A close observation of **Figure 1** shows the physical and practical adaptability of the students to interventional design, which has manifested in the outstanding results observed herein.

Table 2 shows the descriptive statistics of the pre- and post-test. It is observed that the post-test had a better mean score of 60.48 with a standard deviation of 15.83 compared to the pre-test mean score of 40.80 with a standard deviation of 12.66. This indicates that the students had a better conceptual understanding of operations on fractions after the intervention roll out and this was as a result of the use of the manipulatives in the teaching and learning activities.

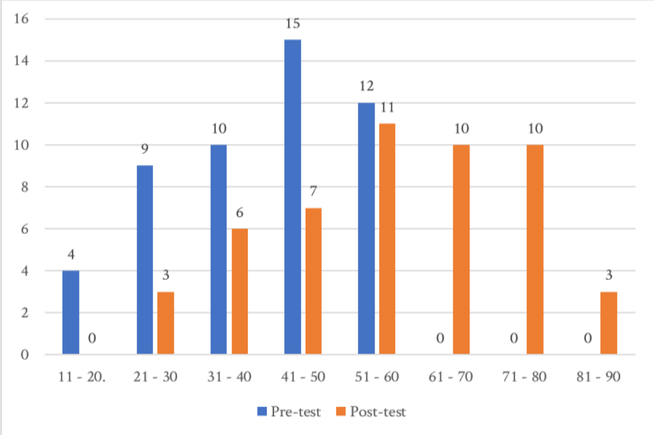

Figure 1. Histogram showing comparison between pre- & post-test scores (Source: Authors)

Table 2. Outputs of descriptive statistics mean, standard deviation, & standard error of mean

		Gender	Pre-test	Post-test
n	Valid	50	50	50
	Missing	0	0	0
Mean		1.40	40.80	60.48
Standard error of mean		.070	1.791	2.239
Median		1.00	40.50	60.00
Mode		1	40[a]	60
Standard deviation		.495	12.664	15.832

Note. [a]Multiple modes exist & the smallest value is shown

Table 3. Paired samples statistics

	Mean	n	Standard deviation
Pre-test score	40.80	50	12.664
Post-test score	60.48	50	15.832

Table 4. Paired samples test

	Paired differences					t	df	Significance	
	Mean difference	Standard deviation	Standard error	95% confidence interval of difference				One-sided	Two-sided
				Lower	Upper				
Pre-/post-test	19.680	8.672	1.226	17.215	22.145	16.04	49	<.001	<.001

Table 5. Mann-Whitney U test for gender differences for pre- & post-test

Variable	Pre-test	Post-test
Total n	50	50
Mann-Whitney U	324.000	356.500
Wilcoxon W	534.000	566.500
Mean rank (male, n=30)	24.70	23.62
Mean rank (female, n=20)	26.70	28.33
Test statistic	324.000	356.500
Standard error	50.426	50.292
Standardized test statistic	.476	1.123
Asymptotic Sig. (2-sided test)	.634	.261

Research Question Two: To What Extent Does Use of Manipulatives to Teach Concept of Operations on Fractions Improve Students' Performance?

To find the answer to the research question two, steps were taken to determine the extent of improvement in students' performance after the intervention and check if there was any statistical significance. As a result, the following hypothesis was tested.

1. **H_0.** There is no statistically significant improvement in students' performance after using the manipulatives.
2. **H_a.** There is a statistically significant improvement in students' performance after using the manipulatives.

Table 3 provides a statistical perspective to bring out clearly, the improvement in the students' performance. **Table 3** shows that the mean performance of the students after the intervention is better than their performance in the control mode. A mean score of 60.48 with a standard deviation of 15.83 in the post-test is far better than the mean and standard deviation of 40.80 and 12.66, respectively, in the pre-test. Therefore, it is prudent to reject the null hypothesis and conclude that there was a statistically significant improvement in the post-test scores of the student.

From **Table 4**, it can be deduced that there was a significant difference between the means of the test conducted before the intervention and the one that was intervened after the training. Since the p-value or the significant value is less than 0.05, it indicates a statistically significant improvement in the students' performance. We, therefore, reject the null hypothesis and conclude that there was a statistically significant improvement in the mean score of the post-test.

Independent-samples Mann-Whitney U test was used to test the distribution of the scores of the pre- and post-test across the male and female students. This was used to examine if there was a significant difference between the male and female students' scores in the pre-test as well as the post-test as indicated in **Table 5** there was no statistically significant difference between the scores obtained by male and female students in the pre-test (male: mean rank=24.70, n=30; female: mean rank=26.70, n=20; U=324.00, Z=0.476, $p=0.634>0.05$). Similarly, the difference between the male and female students' scores in the post-test was not statistically significant (male: mean rank=23.62, n=30; female: mean rank=28.33, n=20; U=356.50, Z=1.123, $p=0.261>0.05$).

From **Table 6**, we reject the null hypothesis and conclude that there was no statistically significant difference between the scores of the males and their female counterparts in the pre-test. Much the same way we reject the null hypothesis and conclude that no significant difference was found between the scores the male students obtained in the post-test and that of the female students.

DISCUSSION

Research question one was to find out the impact of the use of the manipulatives in teaching the concept of operations on fractions. As analyzed, the results from the post-test have proven that the manipulatives positively impact students' conceptual understanding. This goes a long way to confirm the recommendations made by modern researchers (Adendorff et al., 2018; Andamon & Tan, 2018; Bartolini & Martignone, 2020; Sulistyaningsih et al., 2017). Therefore, effective use of manipulatives enhances students' conceptual understanding and must therefore be encouraged. This is because students get the

Table 6. Hypothesis test summary

	Null hypothesis	Test	Sig.	Decision
1	Distribution of pre-test is same across categories of gender.	Independent-samples Mann-Whitney U test	.634	Retain null hypothesis
2	Distribution of post-test is same across categories of gender.	Independent-samples Mann-Whitney U test	.261	Retain null hypothesis

Note. Asymptotic significances are displayed & the significance level is .050

opportunity to interact with materials and with their peers to find solutions to problems by themselves. They tend to understand the concepts better, retain the knowledge acquired, and apply it to new situations.

In respect of the research question two, which sort to find out the extent to which the use of the manipulatives can enhance students' performance, the analyses have proven beyond doubt that the effect size was good enough; this makes it clear enough to conclude that the statistical significance in the students' performance was appreciable. We can, therefore, prudently conclude that using the manipulatives in teaching the concept of operation on fractions is indeed grounded, as observed by various researchers (Adendorff et al., 2018; Cramer & Henry, 2002; Shaw, 2002).

The comparison of students' pre- and post-test scores with respect to gender showed that both the male and female students in the class were at the same level in terms of their conceptual understanding of operations on fractions. Neither category of students could outperform the other. This revelation confirms scores of work done by some researchers who had similar results (Ajai & Imoko, 2015; Alkhateeb, 2001; Armah et al., 2020)

Implications

Ghana's educational system seeks to ensure students' academic growth and a conceptual understanding of concepts especially in the fields of mathematics and science. This can be achieved by way of using a multidisciplinary approach in teaching in order to meet the needs of diverse students. Teaching methods that make use of concrete teaching aids such as manipulatives should be employed when teaching abstract concepts to help in students' conceptualisation. The is re-echoed by NCTM (2000), which encourages the use of manipulatives in teaching abstract concepts across grade levels. In light of this the mathematics curriculum has to be reviewed to incorporate activities that make concepts realistic than abstract in the teaching and learning activities. Furthermore, the goal of teachers should be to make use of these concrete, hands-on manipulatives during teaching and learning of mathematics to facilitate students' academic growth and ultimately achieve better student performance.

CONCLUSIONS

As per the study findings, the researcher wishes to conclude that heads of junior high schools encourage manipulatives in teaching mathematics, especially fractions. By so doing, the mathematics lessons become so realistic and practical. Teachers should also try to have a conceptual understanding of every topic before they attempt teaching it. This will help them to relate the lesson to real-life activities.

Learners, on the hand, will appreciate the subject very well if they know the impact and implication of mathematics in their daily activities. As a result, the school authorities must ensure that teaching and learning materials are available for teachers to enhance smooth interaction.

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REFERENCES

Abudu, A. M., & Mensah, M. A. (2016). Basic schoolteachers' perceptions about curriculum design in Ghana. *Journal of Education and Practice, 7*(19), 21-29.

Adendorff, S. A., Mntunjani, L. M., & Siyepu, S. W. (2018). Foundation phase teachers' use of manipulatives to teach number concepts: A critical analysis. *South African Journal of Childhood Education, 8*(1), 1-9. https://doi.org/10.4102/sajce.v8i1.495

Adu, E., Mereku, D. K., Assuah, C. K., & Okpoti, C. A. (2017). Effect of multimedia courseware with cooperative learning on senior high school students' proficiency in solving linear equation word problems. *African Journal of Educational Studies in Mathematics and Sciences, 13*, 1-11.

Adu-Gyamfi, S., Donkoh, W. J., & Addo, A. A. (2016). Educational reforms in Ghana: Past and present. *Journal of Education and Human Development, 5*(3), 158-172.

Agyei, E., Agamah, D. C., & Entsie, G. (2022). Availability of manipulatives in teaching and learning of mathematics in colleges of education in Ghana. *American Journal of Educational Research, 10*(4), 188-193. https://doi.org/10.12691/education-10-4-5

Ajai, J. T., & Imoko, B. I. (2015). Gender differences in mathematics achievement and retention scores: A case of problem-based learning method. *International Journal of Research in Education and Science, 1*(1), 45-50. https://doi.org/10.21890/ijres.76785

Alkhateeb, H. M. (2001). Gender differences in mathematics achievement among high school students in the United Arab Emirates, 1991-2000. *School Science and Mathematics, 101*(1), 5-9. https://doi.org/10.1111/j.1949-8594.2001.tb18184.x

Alolga, J. A., & Essel, H. B. (n. d.). Progressive mathematics initiative (PMI): An innovative approach to teaching and learning mathematics, evidence from three senior high schools in Ghana. *International Journal of Innovative Science and Research Technology, 6*(5), 1135-1143.

Andamon, J. C., & Tan, D. A. (2018). Conceptual understanding, attitude and performance in mathematics of grade 7 students. *International Journal of Scientific & Technology Research, 07*(08), 96-105.

Armah, S. E., Akayuure, P., & Armah, R. B. (2020). A comparative study of male and female distance learners' mathematics achievement. *Contemporary Mathematics and Science Education, 2*(1), ep21001. https://doi.org/10.30935/conmaths/9288

Baah-Duodu, S., Osei-Buabeng, V., Cornelius, E. F., Hegan, J. E., & Nabie, M. J. (2020). Review of literature on teaching and learning geometry and measurement: A case of Ghanaian standards based mathematics curriculum. *International Journal of Advances in Scientific Research and Engineering, 6*(3), 103-124. https://doi.org/10.31695/IJASRE.2020.33766

Barnes, H. (2005). The theory of realistic mathematics education as a theoretical framework for teaching low attainers in mathematics. *Pythagoras, 0*(61), 42-57. https://doi.org/10.4102/pythagoras.v0i61.120

Bartolini, M. G., & Martignone, F. (2020). Manipulatives in Mmathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 487-494). Springer. https://doi.org/10.1007/978-3-030-15789-0_93

Berger, N., Mackenzie, E., & Holmes, K. (2020). Positive attitudes towards mathematics and science are mutually beneficial for student achievement: A latent profile analysis of TIMSS 2015. *The Australian Educational Researcher, 47*(3), 409-444. https://doi.org/10.1007/s13384-020-00379-8

Bingham, T., & Rodriguez, R. C. (2019). Understanding fractions begins with literacy. *Texas Association for Literacy Education Yearbook, 6*, 9-18.

Boakye, B. A. (2019). *Explaining education reforms in Ghana: An institutional and ideational perspective* [PhD thesis, University of Saskatchewan].

Bosson-Amedenu, S. (2017). Pre-SHS students' perception of difficult concepts in junior high school mathematics curriculum in Ghana. *Asian Research Journal of Mathematics, 3*(2), 1-11. https://doi.org/10.9734/ARJOM/2017/32329

Bouck, E. C., Bassette, L., Shurr, J., Park, J., Kerr, J., & Whorley, A. (2017). Teaching equivalent fractions to secondary students with disabilities via the virtual–representational–abstract instructional sequence. *Journal of Special Education Technology, 32*(4), 220-231. https://doi.org/10.1177/0162643417727291

Caglayan, G. (2019). Theory of polygonal numbers with Cuisenaire rods manipulatives: Understanding Theon of Smyrna's arithmetic in a history of mathematics classroom. *British Journal for the History of Mathematics, 34*(1), 12-22. https://doi.org/10.1080/17498430.2018.1539375

Cain, V. (2021). *Schools and screens: A watchful history*. The MIT Press. https://doi.org/10.7551/mitpress/12125.001.0001

Cain-Caston, M. (1996). Manipulative queen. *Journal of Instructional Psychology, 23*(4), 270.

Chappell, M. F., & Strutchens, M. E. (2001). Creating connections: Promoting algebraic thinking with concrete models. *Mathematics Teaching in the Middle School, 7*(1), 20-25. https://doi.org/10.5951/MTMS.7.1.0020

Clements, D. H. (1999). Subitizing: What is it? Why teach it? *Teaching Children Mathematics*, 5(7), 400-405. https://doi.org/10.5951/TCM.5.7.0400

Clements, D. H., Sarama, J., & Joswick, C. (2022). Learning and teaching geometry in early childhood. In *Special issues in early childhood mathematics education research* (pp. 95-131). Brill. https://doi.org/10.1163/9789004510685_005

Collom, G. D. (2021). A quasi-experimental investigation of Tennessee promise and career and technical education postsecondary enrollment responses. *Postsecondary Education Research Center*. https://trace.tennessee.edu/cgi/viewcontent.cgi?article=1004&context=utk_edleadpubs

Cramer, K., & Henry, A. (2002). Using manipulative models to build number sense for addition of fractions. In B. Litwiller, & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions: 2002 yearbook* (pp. 41-48). National Council of Teachers of Mathematics.

Cramer, K., Monson, D., Whitney, S., Leavitt, S., & Wyberg, T. (2010). Dividing fractions and problem solving. *Mathematics Teaching in the Middle School, 15*(6), 338-346. https://doi.org/10.5951/MTMS.15.6.0338

Dowling, A. P., Wright, K., & Bailey, K. (2018). Academic collaboration for experiential learning: Perspectives on using archival collections and information literacy in history education. *College & Research Libraries News, 79*(6), 323. https://doi.org/10.5860/crln.79.6.323

Etikan, I., & Bala, K. (2017). Sampling and sampling methods. *Biometrics & Biostatistics International Journal, 5*(6), 00149. https://doi.org/10.15406/bbij.2017.05.00149

Heddens, J. W. (1986). Bridging the gap between the concrete and the abstract. *The Arithmetic Teacher, 33*(6), 14-17. https://doi.org/10.5951/AT.33.6.0014

Heuser, D. (2000). Mathematics workshop: Mathematics class becomes learner centered. *Teaching Children Mathematics, 6*(5), 288-295. https://doi.org/10.5951/TCM.6.5.0288

Hodges, T. E., Cady, J., & Collins, R. L. (2008). Fraction representation: The not-so-common denominator among textbooks. *Mathematics Teaching in the Middle School, 14*(2), 78-84. https://doi.org/10.5951/MTMS.14.2.0078

Jimenez, B. A., & Stanger, C. (2017). Math manipulatives for students with severe intellectual disability: A survey of special education teachers. *Research, Advocacy, and Practice for Complex and Chronic Conditions, 36*(1), 1-12.https://doi.org/10.14434/pders.v36i1.22172

Johnson-Smith, L. (2022). Creatively cultivating a culturally-responsive mathematics classroom. In A. G. raj (Ed.), *Creativity as progressive pedagogy: Examinations into culture, performance, and challenges* (pp. 268-295). IGI Global. https://doi.org/10.4018/978-1-7998-8287-9.ch013

Kontas, H. (2016). The effect of manipulatives on mathematics achievement and attitudes of secondary school students. *Journal of Education and Learning, 5*(3), 10. https://doi.org/10.5539/jel.v5n3p10

Larbi, E., & Mavis, O. (2016). The use of manipulatives in mathematics education. *Journal of Education and Practice, 7*(36), 53-61.

Lee, M. Y., Choy, B. H., & Mizzi, A. (2021). Exploring the introduction of fractions in Germany, Singapore, and South Korea mathematics textbooks. *Research in Mathematical Education, 24*(2), 111-130.

Maldonado, S. I., Mosqueda, E., Capraro, R. M., & Capraro, M. M. (2018). Language minority students' mathematics achievement in urban schools: Coursework, race-ethnicity, and English-language proficiency. *Penn GSE Perspectives on Urban Education, 15*(1), 1.

Marsh, L. G., & Cooke, N. L. (1996). The effects of using manipulatives in teaching math problem solving to students with learning disabilities. *Learning Disabilities Research and Practice, 11*(1), 58-65.

McNeil, N., & Jarvin, L. (2007). When theories don't add up: Disentangling the manipulatives debate. *Theory into Practice, 46*(4), 309-316. https://doi.org/10.1080/00405840701593899

Mereku, D. K. (2004). *Mathematics curriculum implementation in Ghana.* Danjoe Production.

Mereku, D. K. (2010). Five decades of school mathematics in Ghana. *Mathematics Connection, 9*(8), 73-86. https://doi.org/10.4314/mc.v9i1.61558

Moch, P. L. (2002). Manipulatives work! *The Educational Forum, 66*(1), 81-87. https://doi.org/10.1080/00131720108984802

MOE, NaCCA. (2019). *Ministry of Education Ghana new curriculum.* National Council for Curriculum Assessment.

Moore, S. D. (2014). Why teach mathematics with manipulatives? *ETA Hand2mind.* https://webringlearningtolife.com/media/contentmanager/content/Why_Teach_Math_with_Manips.pdf

Moyer, P. S. (2001). Are we having fun yet? How teachers use manipulatives to teach mathematics. *Educational Studies in Mathematics, 47*(2), 175-197. https://doi.org/10.1023/A:1014596316942

Nabie, M. J., Akayuure, P., & Sofo, S. (2013). Integrating problem solving and investigations in mathematics: Ghanaian teachers' assessment practices. *International Journal of Humanities and Social Science, 3*(15), 46-56.

NCSM (2013). *Improving student achievement in mathematics by using manipulatives with classroom instruction.* National Council of Supervisors of Mathematics.

NCTM. (2000). *Principles and standards for school mathematics.* National Council of Teachers of Mathematics.

Niemi, D. (1996). Assessing conceptual understanding in mathematics: Representations, problem solutions, justifications, and explanations. *The Journal of Educational Research, 89*(6), 351-363. https://doi.org/10.1080/00220671.1996.9941339

Nikiforidou, Z. (2019). Probabilities and preschoolers: Do tangible versus virtual manipulatives, sample space, and repetition matter? *Early Childhood Education Journal, 47*(6), 769-777. https://doi.org/10.1007/s10643-019-00964-2

OECD. (2017). *OECD science, technology and innovation outlook 2016.* OECD Publishing. https://doi.org/10.1787/sti_in_outlook-2016-en

Ross, R., & Kurtz, R. (1993). Making manipulatives work: A strategy for success. *The Arithmetic Teacher, 40*(5), 254-257. https://doi.org/10.5951/AT.40.5.0254

Ruzic, R., & O'Connell, K. (2001). *Manipulatives.* National Center of Accessing the General Curriculum.

Sebesta, L. M., & Martin, S. R. M. (2004). Fractions: Building a foundation with concrete manipulatives. *Illinois Schools Journal, 83*(2), 3-23.

Shadish, W. R., Cook, T. D., & Campbell, D. T. (2002). *Experimental and quasi-experimental designs for generalized causal inference.* Houghton Mifflin,.

Shaw, J. M. (2002). *Manipulatives enhance the learning of mathematics.*

Stein, M. K., & Bovalino, J. W. (2001). Reflections on practice: Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School, 6*(6), 356-359. https://doi.org/10.5951/MTMS.6.6.0356

Sulistyaningsih, D., Mawarsari, V. D., & Hidayah, I. (2017). Manipulatives implementation for supporting learning of mathematics for prospective teachers. *Journal of Physics: Conference Series, 824*(1), 012047. https://doi.org/10.1088/1742-6596/824/1/012047

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